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BASIC EQUATIONS OF TURBULENCE IN GAS–LIQUID TWO-PHASE FLOW

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Abstract—Basic equations of turbulence in gas–liquid two-phase flow were derived. Based on the local instant formulation of two-phase flow and its averaging, the conservation equations of mass and momentum were obtained for the fluctuating part of the velocity. From these equations, the conservation equations of turbulent energy and turbulent dissipation were derived. In the equation of turbulent energy, interfacial terms were composed of turbulence production due to the relative velocity between the two phases and the exchange between turbulent and surface energy. In the equation of turbulent dissipation, many interfacial terms appear. Some discussions on these interfacial terms and their physical aspects are presented.

Key Words: basic equation, gas–liquid two-phase flow, turbulent energy, turbulent dissipation, interfacial transfer terms

1. INTRODUCTION

Recent advances in two-phase flow researches are remarkable and many things have been clarified about various phenomena in two-phase flow. However, of course, there are still many more things to be studied in order to achieve sufficient understanding and satisfactory predictions about two-phase flow. In particular, the microscopic structures of two-phase flow, such as velocity and phase distributions, interfacial structures and turbulence phenomena, are quite important topics and much effort has been made in these research areas in recent years. This is partly due to scientific interest in the physical phenomena of two-phase flow and partly due to industrial demands for more precise predictions of two-phase flow behavior in various industrial devices.

Turbulence is one of the most important key parameters which determine microscopic structures in two-phase flow. In the area of gas–liquid two-phase flow, several experimental researches have been performed on turbulence phenomena (Serizawa 1974; Serizawa *et al.* 1975, 1984; Lance & Bataille 1983; Michiyoshi & Serizawa 1984; Lahey 1987; Ohba & Yuhara 1982; Theofanous & Sullivan 1982; Inoue *et al.* 1976). As for analytical works, some attempts have been made to predict velocity and phase distributions based on phenomenological modellings of two-phase flow turbulence (Sato *et al.* 1981; Michiyoshi & Serizawa 1984; Drew & Lahey 1981). These modellings are based on partial modifications of single-phase flow turbulence, i.e. introducing bubble-induced turbulence in addition to single-phase flow turbulence. These methods have had certain successes in the prediction of velocity and phase distributions but the applicabilities are limited because their turbulence correlations are strongly dependent on experimental data. Furthermore, the experimental results show that in gas–liquid two-phase flow in certain conditions, the turbulence intensities are smaller than those of single-phase flow (Serizawa 1974; Serizawa *et al.* 1975, 1984; Ohba & Yuhara 1982). This means that two-phase flow turbulence is not the simple sum of bubble-induced turbulence and single-phase flow turbulence.

Therefore, more general methods are needed in order to predict gas–liquid two-phase flow turbulence. In single-phase flow, several sophisticated modellings on turbulence have already been developed. Among these, the k–ϵ model is one of the most popular. In this model, using the conservation equations of turbulent energy and turbulent dissipation, hydrodynamic phenomena in single-phase flow under various situations are analyzed.

In view of this, in this paper, the basic conservation equations of turbulence in gas–liquid two-phase flow are derived based on the local instant and averaged formulations of two-phase flow (Kataoka 1986). However, in gas–liquid two-phase flow, such a methodology has not been established. Even basic equations for turbulence have not been given rigorously yet. The reader may find this surprising compared with recent advances in turbulence analyses in single-phase flow and gas–solid and liquid–solid two-phase flow (Besnard & Harlow 1988; Hetsroni 1989). This is due to complicated configurations and motions of the gas–liquid interface. Some attempts to apply the k–ϵ model to gas–liquid two-phase flow have been made (Lahey 1987). However, the basic equations used in the analysis were based on single-phase flow basic equations of turbulence and details of the interfacial transfer terms were not given. An extension of the single-phase k–ϵ model to gas–liquid two-phase flow needs to be examined carefully based on rigorous basic equations of turbulence in gas–liquid two-phase flow, particularly taking into consideration the interfacial transfer terms of turbulence. In these basic equations of turbulence, there are many interfacial transfer terms. Some discussions and approximations are made on these interfacial transfer terms.

It will be possible to derive various kinds of conservation equations for various turbulence quantities of gas–liquid two-phase flow. Among these, in this paper, the conservation equations of turbulent kinetic energy and turbulent dissipation are derived. The turbulent kinetic energy is the most fundamental quantity in turbulence and its conservation equation is quite important in analyzing turbulence. On the other hand, the conservation equation of turbulent dissipation is merely one of several choices in order to close the turbulence equations. The equation itself already has some models and it is less important and of less general value compared with the turbulent kinetic energy equation. However, at the present stage of turbulence analyses in gas–liquid two-phase flow, only an attempt was made to apply the single-phase k–ϵ model to gas–liquid two-phase flow. Therefore, in order to evaluate the validity and limitations of the k–ϵ model, the rigorous formulation of turbulent dissipation in gas–liquid two-phase flow is considered to be necessary.

Other types of conservation equations of other turbulent quantities in gas–liquid two-phase flow can be obtained in similar way to the turbulent kinetic energy and turbulent dissipation equations. These rigorous conservation equations can serve as bases for developing the methodology of turbulent analyses in gas–liquid two-phase flow.

2. LOCAL INSTANT FORMULATION OF TWO-PHASE FLOW AND ITS AVERAGING

In order to obtain the rigorous formulation of two-phase flow turbulence, one needs to establish the basic equations which describe locally and instantaneously the conservations of mass, momentum and energy in gas–liquid two-phase flow. There are several types of local instant formulations of two-phase flow (Ishii 1975; Delhaye 1968; Bouré 1973; Kataoka 1986).

Here, we use the local instant formulation developed previously by one of the authors (Kataoka 1986). In this formulation, local instant conservation equations of mass, momentum and energy are given in field equations which are uniquely defined in all time and space domains under consideration. This type of formulation is particularly convenient in deriving the conservation equations of turbulence.

In this formulation, local instant mass, momentum and energy conservation equations are given by (Kataoka 1986):

mass,

$$\frac{\partial}{\partial t}(\phi_k \rho_k) + \operatorname{div}(\phi_k \rho_k \mathbf{v}_k) = -\rho_{ki}(\mathbf{v}_{ki} - \mathbf{v}_i)\cdot \mathbf{n}_{ki} a_i \qquad (k = 1, 2); \tag{1}$$

momentum,

$$\frac{\partial}{\partial t}(\phi_k \rho_k \mathbf{v}_k) + \operatorname{div}(\phi_k \rho_k \mathbf{v}_k \mathbf{v}_k) = -\operatorname{grad}(\phi_k P_k) + \operatorname{div}(\phi_k \tau_k) + \phi_k \rho_k \mathbf{F}_k + [-\{\rho_{ki}\mathbf{v}_{ki}\cdot(\mathbf{v}_{ki} - \mathbf{v}_i)\mathbf{n}_{ki}\} - P_{ki}\mathbf{n}_{ki} + \mathbf{n}_{ki}\tau_{ki}]a_i \qquad (k = 1, 2); \tag{2}$$

and

energy,

$$\frac{\partial}{\partial t}\{\phi_k\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)\}+\operatorname{div}\{\phi_k\rho_k(U_k+\tfrac{1}{2}\mathbf{v}_k^2)\mathbf{v}_k\}$$
$$=-\operatorname{div}(\phi_k\mathbf{q}_k)-\operatorname{div}(\phi_kP_k\mathbf{v}_k)+\operatorname{div}(\phi_k\tau_k\cdot\mathbf{v}_k)+\phi_k\rho_k\mathbf{F}_k\cdot\mathbf{v}_k+\phi_kQ_k$$
$$+\{-\rho_{ki}(U_{ki}+\tfrac{1}{2}\mathbf{v}_{ki}^2)(\mathbf{v}_{ki}-\mathbf{v}_i)\cdot\mathbf{n}_{ki}-\mathbf{n}_{ki}\cdot\mathbf{q}_{ki}-P_{ki}\mathbf{v}_{ki}\cdot\mathbf{n}_{ki}+\mathbf{n}_{ki}\cdot(\tau_{ki}\cdot\mathbf{v}_{ki})\}a_i \qquad (k=1,2). \quad [3]$$

Here, ρ_k, $\mathbf{v}_k$, P_k, τ_k, $\mathbf{F}_k$, U_k, $\mathbf{q}_k$ and Q_k are the density, velocity (vector), pressure, stress tensor, external force (vector), internal energy, heat flux (vector) and internal heat generation rate of phase k; $\mathbf{v}_i$ is the velocity vector of the interface and the subscript ki denotes the value of phase k at the interface; and $\mathbf{n}_{ki}$ is the normal outward unit vector of phase k at the interface, as shown in figure 1.

ϕ_k is the characteristic function of phase k, which is defined by

$$\phi_1(x,y,z,t)=h(f(x,y,z,t)) \quad [4]$$

and

$$\phi_2(x,y,z,t)=1-h(f(x,y,z,t)), \quad [5]$$

where $h(w)$ is the Heaviside function, defined by

$$h(w)=1 \quad (w>0)$$
$$=0 \quad (w<0). \quad [6]$$

Here, $f(x,y,z,t)$ is the function which represents an interface, given by

$$f(x,y,z,t)>0 \quad \text{within phase 1}$$
$$f(x,y,z,t)<0 \quad \text{within phase 2}$$
$$f(x,y,z,t)=0 \quad \text{at the interface.} \quad [7]$$

$a_i(x,y,z,t)$ is the local instant interfacial area concentration (interfacial area per unit volume), which is given by (Kataoka *et al.* 1986)

$$a_i(x,y,z,t)=|\operatorname{grad} f|\,\delta(f(x,y,z,t)), \quad [8]$$

where $\delta(w)$ is the delta function, defined by

$$\int_{-\infty}^{\infty}\psi(w)\delta(w-w_0)\,\mathrm{d}w=\psi(w_0). \quad [9]$$

In this paper, as a first step, the conservation equations of turbulence will be derived for the simplest case where both phases can be regarded as incompressible and there is no phase change. In this case, it is sufficient to consider the mass and momentum equations, [1] and [2], only.

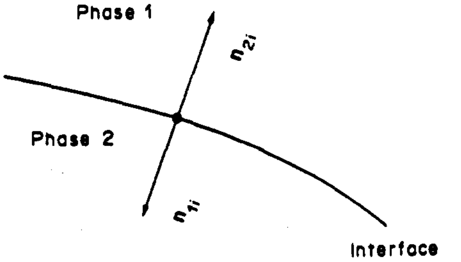

Figure 1. Outward unit normal vector of each phase.

Equations [1] and [2] themselves are rather inconvenient to derive the conservation equations of turbulence. Therefore, one modifies them and changes the notation under the assumptions of incompressibility and no phase change:

mass,

$$\phi_k \frac{\partial v_{k\beta}}{\partial x_\beta} = 0 \qquad (k = 1, 2); \qquad [10]$$

and

momentum,

$$\phi_k \frac{\partial v_{k\alpha}}{\partial t} + \phi_k \frac{\partial}{\partial x_\beta}(v_{k\alpha} v_{k\beta}) = -\phi_k \frac{1}{\rho_k}\frac{\partial P_k}{\partial x_\alpha} + \phi_k \frac{1}{\rho_k}\frac{\partial \tau_{k\alpha\beta}}{\partial x_\beta} + \phi_k F_{k\alpha} \qquad (k = 1, 2). \qquad [11]$$

Here, the subscripts α, β, and γ denote cartesian coordinate components. For example, $v_{k\alpha}$ is an x_α component of velocity of phase k. Einstein's summation rule is applied to these subscripts α, β and γ, while the rule is not applied to subscript k.

In deriving [10] and [11] from [1] and [2], one should be careful in the differentiation of ϕ_k, because it is a discontinuous function, Detailed discussions about this can be found in Kataoka (1986).

The averaged conservation equations of mass and momentum are obtained by appropriately averaging [10] and [11]. There are several averaging methods (time, spatial and statistical). Among these, the most general is statistical averaging (Kataoka & Serizawa 1987). Therefore, in this paper, averaging is in the sense of statistical averaging.

Averaging [10] and [11], one obtains:

averaged mass,

$$\frac{\partial \overline{\overline{v}}_{k\beta}}{\partial x_\beta} = -\frac{1}{\overline{\phi_k}} \overline{v'_{k\beta i} n_{ki\beta} a_i} \qquad (k = 1, 2); \qquad [12]$$

and

averaged momentum,

$$\frac{\partial \overline{\overline{v}}_{k\alpha}}{\partial t} + \frac{\partial}{\partial x_\beta}(\overline{\overline{v}}_{k\alpha}\overline{\overline{v}}_{k\beta}) = -\frac{1}{\rho_k}\frac{\partial \overline{\overline{P}}_k}{\partial x_\alpha} + \frac{1}{\rho_k}\frac{\partial}{\partial x_\beta}(\overline{\overline{\tau}}_{k\alpha\beta} - \overline{\overline{v'_{k\alpha} v'_{k\beta}}}) + \overline{\overline{F}}_{k\alpha} - \frac{\overline{\overline{v}}_{k\alpha}}{\overline{\phi_k}} \overline{v'_{k\beta i} n_{ki\beta} a_i}$$
$$- \frac{1}{\overline{\phi_k}}\frac{1}{\rho_k}\overline{P'_{ki} n_{ki\alpha} a_i} + \frac{1}{\overline{\phi_k}}\frac{1}{\rho_k}\overline{\tau'_{k\alpha\beta i} n_{ki\beta} a_i} + \frac{1}{\overline{\phi_k}}\frac{1}{\rho_k}\overline{\overline{v'_{k\alpha} v'_{k\beta}}}\,\overline{n_{ki\beta} a_i} \qquad (k = 1, 2); \qquad [13]$$

here $\overline{}$ denotes averaging and $\overline{\overline{}}$ denotes the phase-weighted averaging, which is defined by

$$\overline{\overline{A}}_k = \frac{\overline{\phi_k A_k}}{\overline{\phi_k}} \qquad (k = 1, 2), \qquad [14]$$

where A_k is an arbitrary physical quantity of phase k. Fluctuating parts of physical quantities are denoted by $'$, which is defined by

$$A'_k = A_k - \overline{\overline{A}}_k \qquad (k = 1, 2) \qquad [15]$$

and

$$A'_{ki} = A_{ki} - \overline{\overline{A}}_k \qquad (k = 1, 2). \qquad [16]$$

Note that A'_k and its derivatives are not field quantities (physical quantities defined in all time and space domains under consideration). However, when they are multiplied by ϕ_k, i.e.

$$\phi_k A'_k \quad \text{and} \quad \phi_k \frac{\partial A'_k}{\partial \xi} \qquad (\xi\text{—time or space coordinate}) \qquad (k = 1, 2),$$

they become field quantities. Similarly A'_{ki} is not a field quantity but $A'_{ki} a_i$ becomes a field quantity.

3. CONSERVATION EQUATIONS OF FLUCTUATING TERMS

From the local instant and averaged formulations of gas–liquid two-phase flow, as described in the previous section, one can derive the conservation equations of fluctuating terms which are needed to derive the conservation equations of turbulent energy and dissipation.

The mass conservation equation for fluctuating terms is obtained by substracting [12] multiplied by ϕ_k from [10]:

$$\phi_k \frac{\partial v'_{k\beta}}{\partial x_\beta} = \frac{\phi_k}{\overline{\phi_k}} \overline{v'_{k\beta i} n_{ki\beta} a_i} \qquad (k = 1, 2). \qquad [17]$$

This equation is also a field equation because the terms

$$\phi_k \frac{\partial v'_{k\beta}}{\partial x_\beta} \quad \text{and} \quad \overline{v'_{k\beta i} n_{ki\beta} a_i} \qquad (k = 1,2)$$

are defined in all time and space domains under consideration.

Similarly, the momentum conservation equation for fluctuating terms is obtained by subtracting [13] multiplied by ϕ_k from [11]:

$$\phi_k \frac{\partial v'_{k\alpha}}{\partial t} + \phi_k \frac{\partial}{\partial x_\beta}(v'_{k\alpha} v'_{k\beta} + v'_{k\alpha}\bar{\bar{v}}_{k\beta} + v'_{k\beta}\bar{\bar{v}}_{k\alpha})$$

$$= -\phi_k \frac{1}{\rho_k}\frac{\partial P'_k}{\partial x_\alpha} + \phi_k \frac{1}{\rho_k}\frac{\partial}{\partial x_\beta}(\tau'_{k\alpha\beta} + \overline{\overline{v'_{k\alpha} v'_{k\beta}}}) + \phi_k F'_{k\alpha} + \frac{\phi_k}{\overline{\phi_k}} \bar{\bar{v}}_{k\alpha} \overline{v'_{k\beta i} n_{ki\beta} a_i}$$

$$+ \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{P'_{ki} n_{ki\alpha} a_i} - \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{\tau'_{k\alpha\beta i} n_{ki\beta} a_i} - \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{\overline{v'_{k\alpha} v'_{k\beta}}}\, \overline{n_{ki\beta} a_i} \qquad (k = 1, 2). \qquad [18]$$

4. CONSERVATION EQUATION OF TURBULENT ENERGY

The turbulent kinetic energy of phase k is represented by

$$\rho_k v'^2_{k\alpha} \qquad (k = 1, 2).$$

Since incompressibility of both phases is assumed here, it is sufficient to consider the quantity $v'^2_{k\alpha}$. The conservation equation of this quantity (the conservation equation of local instant turbulent energy) is obtained by multiplying [18] by $v'_{k\alpha}$ and modifying using [17]:

$$\phi_k \frac{\partial}{\partial t}(\tfrac{1}{2} v'^2_{k\alpha}) + \phi_k \frac{\partial}{\partial x_\beta}(\tfrac{1}{2} v'^2_{k\alpha} \bar{\bar{v}}_{k\beta})$$

$$= -\phi_k \frac{1}{\rho_k}\frac{\partial}{\partial x_\alpha}(P'_k v'_{k\alpha}) - \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} P'_k \overline{v'_{k\alpha i} n_{ki\alpha} a_i} + \phi_k \frac{1}{\rho_k} \frac{\partial}{\partial x_\beta}(\tau'_{k\alpha\beta} v'_{k\alpha} + \overline{\overline{v'_{k\alpha} v'_{k\beta}}} v'_{k\alpha})$$

$$- \phi_k \frac{1}{\rho_k}(\tau'_{k\alpha\beta} + \overline{\overline{v'_{k\alpha} v'_{k\beta}}}) \frac{\partial v'_{k\alpha}}{\partial x_\beta} + \phi_k F'_{k\alpha} v'_{k\alpha} - \phi_k \frac{\partial}{\partial x_\beta}(\tfrac{1}{2} v'^2_{k\alpha} v'_{k\beta})$$

$$- \phi_k v'_{k\alpha} v'_{k\beta} \frac{\partial \bar{\bar{v}}_{k\alpha}}{\partial x_\beta} + \frac{\phi_k}{\overline{\phi_k}} \frac{1}{\rho_k} \overline{v'_{k\alpha}(P'_{ki} n_{ki\alpha} - \tau'_{k\alpha\beta i} n_{ki\beta} - \overline{\overline{v'_{k\alpha} v'_{k\beta}}} n_{ki\beta}) a_i} \qquad (k = 1, 2). \qquad [19]$$

Averaging [19] and modifying in view of

$$\overline{\phi_k A'_k} = 0, \qquad [20]$$

one obtains the conservation equation of averaged turbulent energy, which is of practical importance:

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{v'^2_{k\alpha}}}\right)+\frac{\partial}{\partial x_\beta}\left(\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{v'^2_{k\alpha}v_{k\beta}}}\right)$$

$$=-\frac{1}{\rho_k}\frac{\partial}{\partial x_\alpha}\left(\overline{\phi_k}\,\overline{\overline{P'_k v'_{k\alpha}}}\right)-\frac{\partial}{\partial x_\beta}\left(\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{v'^2_{k\alpha}v'_{k\beta}}}\right)+\frac{1}{\rho_k}\frac{\partial}{\partial x_\beta}\left(\overline{\phi_k}\,\overline{\overline{\tau'_{k\alpha\beta}v'_{k\alpha}}}\right)-\frac{1}{\rho_k}\overline{\phi_k}\,\overline{\overline{\tau'_{k\alpha\beta}\frac{\partial v'_{k\alpha}}{\partial x_\beta}}}$$

$$-\overline{\phi_k}\,\overline{\overline{v'_{k\alpha}v'_{k\beta}}}\frac{\partial\overline{\overline{v}}_{k\alpha}}{\partial x_\beta}+\overline{\phi_k}\,\overline{\overline{F'_{k\alpha}v'_{k\alpha}}}-\frac{1}{\rho_k}\overline{P'_{ki}v'_{k\alpha i}n_{ki\alpha}a_i}+\frac{1}{\rho_k}\overline{\tau'_{k\alpha\beta i}v'_{k\alpha i}n_{ki\beta}a_i}\qquad (k=1,2). \qquad [21]$$

The first three terms on the r.h.s. of [21] represent the diffusion of turbulent energy. The fourth term represents the turbulent dissipation and the fifth term the turbulent generation. The last two terms, which include interfacial area concentration, represent the interfacial transport of turbulent energy. The physical meanings of these interfacial turbulent transport terms will be made clearer when one considers the sum of the turbulent energies of both phases. Multiplying [21] by ρ_k and making a summation for $k = 1, 2$, one obtains the conservation equation of turbulent energy of a two-phase mixture as a whole:

$$\frac{\partial}{\partial t}\left(\sum_{k=1}^{2}\tfrac{1}{2}\overline{\phi_k}\rho_k\overline{\overline{v'^2_{k\alpha}}}\right)+\frac{\partial}{\partial x_\beta}\left(\sum_{k=1}^{2}\tfrac{1}{2}\overline{\phi_k}\rho_k\overline{\overline{v'^2_{k\alpha}}}\,\overline{\overline{v}}_{k\beta}\right)$$

$$=-\frac{\partial}{\partial x_\alpha}\left(\sum_{k=1}^{2}\overline{\phi_k}\,\overline{\overline{P'_{k\alpha}v'_{k\alpha}}}\right)-\frac{\partial}{\partial x_\beta}\left(\sum_{k=1}^{2}\tfrac{1}{2}\overline{\phi_k}\rho_k\overline{\overline{v'_{k\alpha}v'_{k\beta}}}\right)+\frac{\partial}{\partial x_\beta}\left(\sum_{k=1}^{2}\overline{\phi_k}\,\overline{\overline{\tau'_{k\alpha\beta}v'_{k\alpha}}}\right)-\sum_{k=1}^{2}\left(\overline{\phi_k}\,\overline{\overline{\tau'_{k\alpha\beta}\frac{\partial v'_{k\alpha}}{\partial x_\beta}}}\right)$$

$$-\sum_{k=1}^{2}\overline{\phi_k}\rho_k\overline{\overline{v'_{k\alpha}v'_{k\beta}}}\frac{\partial\overline{\overline{v}}_{k\alpha}}{\partial x_\beta}+\sum_{k=1}^{2}\overline{\phi_k}\rho_k\overline{\overline{F'_{k\alpha}v'_{k\alpha}}}-\sum_{k=1}^{2}\overline{(P'_{ki}v'_{k\alpha i}n_{ki\alpha}a_i)}+\sum_{k=1}^{2}\overline{(\tau'_{k\alpha\beta i}v'_{k\alpha i}n_{ki\beta}a_i)}. \qquad [22]$$

Here, using the definition of fluctuation, [16], the interfacial transport terms can be rewritten as

$$-\sum_{k=1}^{2}\overline{(P'_{ki}v'_{k\alpha i}n_{ki\alpha}a_i)}+\sum_{k=1}^{2}\overline{(\tau'_{k\alpha\beta i}v'_{k\alpha i}n_{ki\beta}a_i)}$$

$$=\sum_{k=1}^{2}\overline{(-P_{ki}n_{ki\alpha}+\tau_{k\alpha\beta i}n_{ki\beta})v_{k\alpha i}a_i}-\sum_{k=1}^{2}\overline{\{-(P_{ki}-\overline{\overline{P}}_k)n_{ki\alpha}+\tau_{k\alpha\beta i}n_{ki\beta}\}a_i}\,\overline{\overline{v}}_{k\alpha}$$

$$+\sum_{k=1}^{2}(\overline{\overline{P}}_{ki}-\overline{\overline{P}}_k)\overline{n_{ki\alpha}a_i}\,\overline{\overline{v}}_{k\alpha}-\sum_{k=1}^{2}(\overline{\overline{P}}_k\overline{v_{k\alpha i}n_{ki\alpha}a_i})+\sum_{k=1}^{2}\overline{\overline{\tau}}_{k\alpha\beta}\overline{v'_{k\alpha i}n_{ki\beta}a_i}, \qquad [23]$$

where $\overline{\overline{P}}_{ki}$ is defined by

$$\overline{\overline{P}}_{ki}=\frac{\overline{P_{ki}a_i}}{\overline{a_i}}\qquad (k=1,2). \qquad [24]$$

The momentum balance at the interface and the definition of interfacial drag force are given by (Ishii 1975)

$$\sum_{k=1}^{2}(-P_{ki}n_{ki\alpha}+\tau_{k\alpha\beta_i}n_{ki\beta})=F_{s\alpha} \qquad [25]$$

and

$$\overline{\{-(P_{ki}-\overline{\overline{P}}_{ki})n_{ki\alpha}+\tau_{k\alpha\beta i}n_{ki\beta}\}a_i}=F_{Dk\alpha}\qquad (k=1,2), \qquad [26]$$

where $F_{s\alpha}$ and $F_{Dk\alpha}$ are the surface tension force at the interface and the interfacial drag force of phase k. From the principle of action and reaction (Newton's third law), the interfacial drag forces satisfy

$$F_{D1\alpha}=-F_{D2\alpha}. \qquad [27]$$

The assumption of no phase change gives

$$v_{1\alpha i} = v_{2\alpha i} = v_{i\alpha}. \qquad [28]$$

From [25]–[28], the first two terms on the r.h.s. of [23] can be rewritten as

$$\sum_{k=1}^{2} \overline{(-P_{ki}n_{ki\alpha} + \tau_{k\alpha\beta i}n_{ki\beta})v_{k\alpha i}a_i} - \sum_{k=1}^{2} \overline{\{-(P_{ki} - \overline{\overline{P}}_{ki})n_{ki\alpha} + \tau_{k\alpha\beta i}n_{ki\beta}\}a_i}\overline{\overline{v}}_{k\alpha}$$
$$= \overline{F_{s\alpha}v_{i\alpha}a_i} - F_{D1\alpha}(\overline{\overline{v}}_{1\alpha} - \overline{\overline{v}}_{2\alpha}). \qquad [29]$$

Here, the term

$$\overline{F_{s\alpha}v_{i\alpha}a_i}$$

represents the work which is done by the surface tension force. This work is equivalent to the reduction of surface energy. Therefore, it is related to the surface energy by

$$\overline{F_s v_i a_i} = -U_s \Gamma_s \overline{a}_i. \qquad [30]$$

where U_s is the surface energy per unit interfacial area (here assumed constant) and Γ_s is the increasing rate of the interfacial area.

The second term on the r.h.s. of [29] is considered to represent the bubble-induced turbulence generation. Since the interfacial drag force is due to the relative velocity between the phases and acts so as to reduce the relative velocity, the interfacial drag force and the relative velocity have opposite directions. This means

$$-F_{D1\alpha}(\overline{\overline{v}}_{1\alpha} - \overline{\overline{v}}_{2\alpha}) > 0. \qquad [31]$$

Therefore, this term is always positive and represents the generation of turbulence.

In addition to the above discussions, one makes some assumptions about the interfacial transport terms of turbulence given by [23]. The assumptions are

$$\overline{\overline{P}}_{ki} = \overline{\overline{P}}_k, \qquad [32]$$

and

$$\overline{v'_{k\alpha i}n_{ki\gamma}a_i} = 0 \qquad (k = 1, 2). \qquad [33]$$

Equation [32] means that averaged pressure at the interface is approximately equal to the bulk averaged pressure. Equation [33] is based on the assumption that the fluctuating motion of the interface is isotropic. Of course, these assumptions need further verification based on experiments and analyses. However, with the present knowledge, it is quite difficult to give more detailed correlations for these quantities and [32] and [33] are considered to be valid as a first approximation.

Using [29]–[33], the interfacial transfer terms given by [23] can be rewritten as

$$-\sum_{k=1}^{2} \overline{(P'_{ki}v'_{k\alpha i}n_{ki\alpha}a_i)} + \sum_{k=1}^{2} \overline{(\tau'_{k\alpha\beta i}v'_{k\alpha i}n_{ki\beta}a_i)} = -U_s\Gamma_s\overline{a}_i - F_{D1\alpha}(\overline{\overline{v}}_{1\alpha} - \overline{\overline{v}}_{2\alpha}) - (\overline{\overline{P}}_1 - \overline{\overline{P}}_2)\frac{\partial\overline{\phi_1}}{\partial t}. \qquad [34]$$

Here, the following relations for the characteristic function of each phase (Kataoka 1986) are used:

$$\frac{\partial\overline{\phi_k}}{\partial t} = \overline{v_{k\alpha i}n_{ki\alpha}a_i} \qquad (k = 1, 2) \qquad [35]$$

and

$$\phi_1 + \phi_2 = 1. \qquad [36]$$

Equation [22] coupled with [34] gives the most practical conservation equation of turbulence energy. However, in order to make a practical calculation of turbulence, further approximations are necessary. In [22], the liquid-phase turbulence energy is considered to be much larger than the gas-phase turbulent energy due to the large difference in the densities of both phases in ordinary combinations of gas and liquid (air–water etc.). Furthermore, the balance equation of the interfacial area concentration (Kataoka 1986) gives

$$U_s\Gamma_s\overline{a}_i = \frac{\partial}{\partial t}(U_s\overline{a}_i) + \frac{\partial}{\partial x_\beta}(U_s\overline{a}_i\overline{\overline{v}}_{i\beta}). \qquad [37]$$

From the above-mentioned approximations and relations, [22] can be rewritten as

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\overline{\phi_1}\rho_1\overline{\overline{v_{1\alpha}'^2}} + U_s\overline{a_i}\right) + \frac{\partial}{\partial x_\beta}\left\{\left(\tfrac{1}{2}\overline{\phi_1}\rho_1\overline{\overline{v_{1\alpha}'^2}} + U_s\overline{a_i}\right)\overline{\overline{v}}_{1\beta}\right\}$$

$$= -\frac{\partial}{\partial x_\alpha}\left(\overline{\phi_1}\,\overline{\overline{P_1'v_{1\alpha}'}}\right) - \frac{\partial}{\partial x_\beta}\left(\tfrac{1}{2}\overline{\phi_1}\rho_1\overline{\overline{v_{1\alpha}'^2 v_{1\beta}'}}\right) + \frac{\partial}{\partial x_\beta}\left(\overline{\phi_1}\,\overline{\overline{\tau_{1\alpha\beta}'v_\alpha'}}\right) - \overline{\phi_1}\,\overline{\overline{\tau_{1\alpha\beta}'\frac{\partial v_{1\alpha}'}{\partial x_\beta}}} - \overline{\phi_1}\rho_1\overline{\overline{v_{1\alpha}'v_{1\beta}'}}\frac{\partial \overline{\overline{v}}_{1\alpha}}{\partial x_\beta}$$

$$+ \overline{\phi_1}\rho_1\overline{\overline{F_{1\alpha}'v_{1\alpha}'}} - F_{D1\alpha}(\overline{\overline{v}}_{1\alpha} - \overline{\overline{v}}_{2\alpha}) - (\overline{\overline{P}}_1 - \overline{\overline{P}}_2)\frac{\partial \overline{\phi_1}}{\partial t}. \qquad [38]$$

Here, the liquid phase is chosen as phase 1 and the averaged interfacial velocity is approximated by the averaged velocity of the liquid phase, i.e.

$$\overline{\overline{v}}_{i\beta} = \overline{\overline{v}}_{1\beta}. \qquad [39]$$

It should be noted that in [38], surface energy terms (effects of surface tension) appear, while in the two-fluid formulation, [21], they do not. This is due to the fact that [38] is the conservation equation of turbulent kinetic energy as a two-phase mixture (though gas-phase turbulence is almost neglected) and the surface tension effects are a result of the summation of both phase equations along with the interfacial momentum balance, given by [25].

Equation [38] has several characteristic terms in gas–liquid two-phase flow. One is bubble-induced (or drag-induced) turbulence generation terms, given by

$$-F_{D1\alpha}(\overline{\overline{v}}_{1\alpha} - \overline{\overline{v}}_{2\alpha}) > 0.$$

Similar terms are reported to appear in particulate flow turbulence (Besnard & Harlow 1988). This term means that in gas–liquid two-phase flow, a part of the turbulence is generated by the relative motion between the gas and the liquid. Naturally, the scale of this turbulence is larger than shear-generated turbulence. For example, it is comparable to bubble size in bubbly flow (several millimeters). Therefore, the turbulent kinetic energy of gas–liquid two-phase flow includes a much wider range of wavelengths for turbulent eddies than single-phase flow. This gives rise to the problem of treating such a wide range of length scale of turbulence in one physical quantity, i.e. turbulent kinetic energy, and analyzing it in a single field equation. To answer this problem, a great deal more knowledge is required concerning the similarities and dissimilarities between bubble-induced and shear-induced turbulence.

This turbulence generation term is mainly dependent on the velocity difference between the two phases (of course, it is related to turbulence itself through the drag coefficient of bubbles etc.). Therefore, it may cause turbulence in the liquid phase where originally turbulence did not exist. One can see a prominent example in the case where gas bubbles are injected into a stagnant liquid.

Another characteristic feature in [38] is the last term on the r.h.s., given by

$$-(\overline{\overline{P}}_1 - \overline{\overline{P}}_2)\frac{\partial \overline{\phi_1}}{\partial t}.$$

It may appear strange that this term appears in the turbulent kinetic energy equation since it is assumed that both phases are incompressible and there is no phase change. In view of the derivation process of [34], the physical meaning of this term is considered to be related to voidage wave propagation in gas–liquid two-phase flow. As shown in [12] and [17], in gas–liquid two-phase flow, the divergences of the mean and fluctuating velocities are not zero, even when incompressibility of both phases and no phase change are assumed. This is due to the fact that volume fractions of both phases are changing in time and space. In order to make this clearer, the averaged mass conservation equation can be written in another way (Ishii 1975),

$$\frac{\partial \overline{\phi_k}}{\partial t} + \overline{\overline{v}}_{k\beta}\frac{\partial \overline{\phi_k}}{\partial x_\beta} + \overline{\phi_k}\frac{\partial \overline{\overline{v}}_{k\beta}}{\partial x_\beta} = 0 \qquad (k = 1, 2), \qquad [40]$$

for incompressible flow without phase change. Comparing [39] with [12], one obtains

$$\frac{\partial \overline{\phi_k}}{\partial t} + \overline{\overline{v}}_{k\beta}\frac{\partial \overline{\phi_k}}{\partial x_\beta} \equiv \frac{1}{\overline{\phi_k}}\overline{v_{k\beta i}'n_{ki\beta}a_i} \qquad (k = 1, 2). \qquad [41]$$

This means the source terms of the conservation equations of the mean and fluctuation velocities are related to the propagation of the volume fractions of both phases. Since the term

$$-(\overline{\overline{P}}_1 - \overline{\overline{P}}_2)\frac{\partial \overline{\phi_1}}{\partial t}$$

comes from the source terms of divergences of the mean and fluctuating velocities, it is considered to reflect the effects of voidage wave propagation on turbulence fields.

In two-dimensional cylindrical coordinates and the steady state, which are of practical importance for analyses in pipe flow, [38] can be given in the following form, under several assumptions similar to single-phase flow turbulence:

$$\frac{\partial}{\partial z}\{(\overline{\phi_1}\rho_1 \mathrm{K}_1 + U_s\overline{a_i})\overline{\overline{v}}_{1z}\} + \frac{\partial}{\partial r}\{(\overline{\phi_1}\rho_1 \mathrm{K}_1 + U_s\overline{a_i})\overline{\overline{v}}_{1r}\} = \frac{1}{r}\frac{\partial}{\partial r}\left(\overline{\phi_1} r \frac{\mu_t}{\sigma_k}\frac{\partial \mathrm{K}_1}{\partial r}\right)$$
$$+ \mu_t\overline{\phi_1}\left(\frac{\partial \overline{\overline{v}}_{1z}}{\partial r}\right)^2 - \rho_1\overline{\phi_1}\epsilon_1 - F_{\mathrm{D}1z}(\overline{\overline{v}}_{1z} - \overline{\overline{v}}_{2z}), \quad [42]$$

where

$$\mathrm{K}_1 = \tfrac{1}{2}\overline{\overline{v'^2_{1\alpha}}}, \quad [43]$$

$$\epsilon_1 = \frac{1}{\rho_1}\overline{\overline{\tau'_{1\alpha\beta}\frac{\partial v'_{1\alpha}}{\partial x_\beta}}} \quad \text{(turbulent dissipation)} \quad [44]$$

and μ_t and σ_k are the turbulent viscosity and a coefficient related to the diffusion of turbulent energy.

As shown in [38] and [42], in the turbulent energy conservation equation of gas–liquid two-phase flow, the volume fraction, interfacial area concentration, interfacial drag force and interfacial energy play predominant roles in determining the turbulent energy distribution. Therefore, detailed knowledge of the phase distribution, interfacial area concentration distribution and the mechanisms of interfacial drag are indispensable in analyzing the turbulent energy distribution in gas–liquid two-phase flow.

In principle, the analyses of turbulence should be carried out based on the conservation equation of each phase, [21], which treats separately the turbulent kinetic energy of each phase. However, at the present stage, we have insufficient knowledge to give separately the constitutive equations of the interfacial transfer terms in [21]. Furthermore, as mentioned above, in ordinary combinations of gas–liquid two-phase flow, the turbulent kinetic energy of the gas phase is much smaller than that of the liquid phase. Therefore, [38] or [42] can be used as the basic equation of turbulent kinetic energy as a first approximation for practical purposes.

5. CONSERVATION EQUATION OF TURBULENT DISSIPATION

The conservation equation of turbulent kinetic energy, derived in the previous section, is the most fundamental equation in analyzing the turbulence phenomena in gas–liquid two-phase flow. However, this equation has many terms which include higher order correlations of velocities, pressure and stress tensors and interfacial turbulent transports. They can be given as constitutive equations based on experimental data. However, for more sophisticated and general analyses, it is desirable to predict some of these terms using the conservation equations themselves. There are many conservation equations for various correlative terms and interfacial transport terms. It is a difficult problem to choose one of them as a basic equation to be coupled with the turbulent kinetic energy equation. For gas–liquid two-phase flow, at present, there is no established methodology in order to close the turbulence equations system. Therefore, in view of the present status of turbulent analyses in gas–liquid two-phase flow, here, as one example, we derive the conservation equation of turbulent dissipation:

$$\epsilon_k = \frac{1}{\rho_k}\overline{\overline{\tau'_{k\alpha\beta}\frac{\partial v'_{k\alpha}}{\partial x_\beta}}} \quad (k = 1, 2). \quad [45]$$

The conservation equations of other correlative terms and interfacial transport terms can be derived in similar ways to those used in this section.

When one assumes that turbulence is isotropic and the fluctuating motions of interfaces are also isotropic, the turbulent dissipation of each phase for a Newtonian fluid is approximated by

$$\epsilon_k = \nu_k \overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\beta}\right)^2}} \qquad (k = 1, 2). \tag{46}$$

Therefore, as the conservation equation of turbulent dissipation in gas–liquid two-phase flow, one has to derive the conservation equation of

$$\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\beta}\right)^2}} \qquad (k = 1, 2).$$

Analogously to the derivation of the conservation equation of turbulent dissipation in single-phase flow, the conservation equation of turbulent dissipation in gas–liquid two-phase flow can be obtained from the differentiation of the local instant and averaged equations of momentum conservation. First, one differentiates the local instant momentum conservation equation [11] with regard to x_γ. From this equation, one subtracts [13] (averaged momentum equation) differentiated with regard to x_γ and multiplied by ϕ_k and, averaging, one finally obtains the conservation equation of averaged turbulent dissipation, which is of practical importance:

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left\{\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2}}\right\} + \frac{\partial}{\partial x_\beta}\left\{\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2}}\,\overline{\overline{v}}_{k\beta}\right\} \\
&= -\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial v'_{k\beta}}{\partial x_\gamma}\frac{\partial v'_{k\alpha}}{\partial x_\beta}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)}} - \overline{\phi_k}\frac{\partial \overline{\overline{v}}_{k\beta}}{\partial x_\gamma}\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\beta}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)}} - \overline{\phi_k}\frac{\partial \overline{\overline{v}}_{k\alpha}}{\partial x_\beta}\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\frac{\partial v'_{k\beta}}{\partial x_\gamma}\right)}} - \overline{\phi_k}\frac{\partial^2 \overline{\overline{v}}_{k\alpha}}{\partial x_\gamma \partial x_\beta}\overline{\overline{\left(v'_{k\beta}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)}} \\
&\quad - \frac{\partial}{\partial x_\beta}\left\{\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2 v'_{k\beta}}}\right\} - \frac{1}{\rho_k}\frac{\partial}{\partial x_\alpha}\left\{\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial P'_k}{\partial x_\gamma}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)}}\right\} + \nu_k\frac{\partial^2}{\partial x_\beta^2}\left\{\tfrac{1}{2}\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2}}\right\} - \nu_k\overline{\phi_k}\,\overline{\overline{\left(\frac{\partial^2 v'_{k\alpha}}{\partial x_\beta \partial x_\gamma}\right)^2}} \\
&\quad + \overline{\phi_k}\,\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\frac{\partial F'_\alpha}{\partial x_\gamma}\right)}} - \frac{1}{\rho_k}\overline{\left(\frac{\partial P'_k}{\partial x_\gamma}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_{\mathrm{i}} n_{ki\alpha}a_{\mathrm{i}}} - \frac{1}{\rho_k}\overline{P'_{ki}n_{ki\gamma}a_{\mathrm{i}}}\frac{\partial}{\partial x_\gamma}\left(\frac{1}{\overline{\phi_k}}\overline{v'_{k\alpha i}n_{ki\alpha}a_{\mathrm{i}}}\right) - \frac{1}{\rho_k}\overline{v'_{k\alpha i}n_{ki\gamma}a_{\mathrm{i}}} \\
&\quad \times \frac{\partial}{\partial x_\gamma}\left\{-\frac{1}{\overline{\phi_k}}\overline{P'_{ki}n_{ki\alpha}a_{\mathrm{i}}} + \frac{1}{\overline{\phi_k}}\overline{\tau'_{k\alpha\beta i}n_{ki\beta}a_{\mathrm{i}}} + \frac{1}{\overline{\phi_k}}\overline{\overline{v'_{k\alpha}v'_{k\beta}}n_{ki\beta}a_{\mathrm{i}}}\right\} \\
&\quad - \nu_k\overline{v'_{k\alpha i}n_{ki\gamma}a_{\mathrm{i}}}\frac{\partial^2}{\partial x_\gamma \partial x_\alpha}\left(\frac{1}{\overline{\phi_k}}\overline{v'_{k\beta i}n_{ki\beta}a_{\mathrm{i}}}\right) + \nu_k\overline{v'_{k\alpha i}n_{ki\gamma}a_{\mathrm{i}}}\frac{\partial^2}{\partial x_\beta \partial x_\gamma}\left(\frac{1}{\overline{\phi_k}}\overline{v'_{k\alpha i}n_{ki\beta}a_{\mathrm{i}}}\right) \\
&\quad + \nu_k\overline{v'_{k\alpha i}n_{ki\gamma}a_{\mathrm{i}}}\frac{\partial^2}{\partial x_\beta \partial x_\gamma}\left(\frac{1}{\overline{\phi_k}}\overline{v'_{k\beta i}n_{ki\alpha}a_{\mathrm{i}}}\right) + \nu_k\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_{\mathrm{i}}^2 n_{ki\beta}a_{\mathrm{i}}}\right\} \\
&\quad + \nu_k\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_{\mathrm{i}}\left(\frac{\partial^2 v'_{k\alpha}}{\partial x_\gamma \partial x_\beta}\right)_{\mathrm{i}} n_{ki\beta}a_{\mathrm{i}}} \qquad (k = 1, 2).
\end{aligned}
\tag{47}
$$

As shown in this equation, in the conservation equation of turbulent dissipation in two-phase flow, there are many terms including interfacial area concentration, i.e. interfacial transport terms. This indicates that the turbulent dissipation in two-phase flow is strongly related to mass, momentum and interfacial energy transport at the interface. Therefore, for general considerations, accurate knowledge of these interfacial transport terms is indispensable.

However, under several assumptions, these interfacial transport terms can be simplified further. First, as described before, when one assumes that the fluctuating motion of the interface is isotropic, the interfacial transport terms which include

$$\overline{v'_{k\alpha i}n_{ki\gamma}a_{\mathrm{i}}} \qquad (k = 1, 2)$$

can be neglected. Furthermore, in view of [12], the term

$$-\frac{1}{\rho_k}\overline{P'_{ki}n_{ki\gamma}a_i}\frac{\partial}{\partial x_\gamma}\left(\frac{1}{\overline{\phi_k}}\overline{v'_{k\alpha i}n_{ki\alpha}a_i}\right) \qquad (k=1,2)$$

can be modified to

$$-\frac{1}{\rho_k}\overline{P'_{ki}n_{ki\gamma}a_i}\left(\frac{\partial^2}{\partial x_\gamma \partial x_\alpha}\overline{\overline{v}}_{k\alpha}\right) \qquad (k=1,2).$$

In ordinary two-phase flow, higher-order derivatives of averaged velocities can be much smaller than first-order derivatives. Therefore, the above-mentioned term can be neglected.

Then, the remaining interfacial transport terms of turbulent dissipation are

$$v_k\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2_i n_{ki\beta}a_i}\right\}, \qquad -\frac{1}{\rho_k}\overline{\left(\frac{\partial P'_k}{\partial x_\gamma}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_i n_{ki\alpha}a_i}$$

and

$$\overline{v_k\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_i\left(\frac{\partial^2 v'_{k\alpha}}{\partial x_\gamma \partial x_\beta}\right)_i n_{ki\beta}a_i}.$$

Among these, the last term contains higher-order derivatives compared with the first two terms and is assumed to be negligible. Then, the first two terms are considered to be the main contribution to the interfacial transport of turbulent dissipation. As shown above, these terms are related to the derivatives of pressure and velocity fluctuations at interface. At present, almost nothing is known regarding these physical quantities, either experimentally or theoretically. Therefore, here, as a first approximation, the interfacial averaged values of the derivatives are assumed to be equal to the phase averaged values, i.e.

$$v_k\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_i n_{ki\beta}a_i}\right\} = v_k\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2}}\,\overline{n_{ki\beta}a_i}\right\}$$

$$= v_k\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}\overline{\overline{\left(\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)^2}}\right\}\frac{\partial\overline{\phi_k}}{\partial x_\beta} \qquad (k=1,2) \qquad [48]$$

and

$$-\frac{1}{\rho_k}\overline{\left(\frac{\partial P'_k}{\partial x_\gamma}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)_i n_{ki\alpha}a_i} = -\frac{1}{\rho_k}\overline{\overline{\left(\frac{\partial P'_k}{\partial x_\gamma}\frac{\partial v'_{k\alpha}}{\partial x_\gamma}\right)}}\frac{\partial\overline{\phi_k}}{\partial x_\alpha} \qquad (k=1,2). \qquad [49]$$

Furthermore, similar to the conservation equation of turbulent energy, when one assumes that the turbulent dissipation of the liquid phase is much larger than that of the gas phase, the conservation equation of turbulent dissipation in two-phase mixture as a whole is given by (corresponding to [38]):

$$\frac{\partial}{\partial t}\left\{\tfrac{1}{2}\overline{\phi_1}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)^2}}\right\} + \frac{\partial}{\partial x_\beta}\left\{\tfrac{1}{2}\overline{\phi_1}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)^2}}\,\overline{\overline{v}}_{1\beta}\right\}$$

$$= -\overline{\phi_1}\overline{\overline{\left(\frac{\partial v'_{1\beta}}{\partial x_\gamma}\frac{\partial v'_{1\alpha}}{\partial x_\beta}\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)}} - \overline{\phi_1}\frac{\partial\overline{\overline{v}}_{1\beta}}{\partial x_\gamma}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\beta}\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)}} - \overline{\phi_1}\frac{\partial\overline{\overline{v}}_{1\alpha}}{\partial x_\beta}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\frac{\partial v'_{1\beta}}{\partial x_\gamma}\right)}} - \frac{\partial}{\partial x_\beta}\left\{\tfrac{1}{2}\overline{\phi_1}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)^2 v'_{1\beta}}}\right\}$$

$$-\frac{1}{\rho_k}\frac{\partial}{\partial x_\alpha}\overline{\phi_1}\overline{\overline{\left(\frac{\partial P'_1}{\partial x_\gamma}\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)}} + v_1\frac{\partial^2}{\partial x_\beta^2}\left\{\tfrac{1}{2}\overline{\phi_1}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)^2}}\right\} - v_1\overline{\phi_1}\overline{\overline{\left(\frac{\partial^2 v'_{1\alpha}}{\partial x_\gamma \partial x_\gamma}\right)^2}} + \overline{\phi_1}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\frac{\partial F'_{1\alpha}}{\partial x_\gamma}\right)}}$$

$$-\frac{1}{\rho_k}\overline{\overline{\left(\frac{\partial P'_1}{\partial x_\gamma}\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)}}\frac{\partial\overline{\phi_1}}{\partial x_\alpha} + v_1\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}\overline{\overline{\left(\frac{\partial v'_{1\alpha}}{\partial x_\gamma}\right)^2}}\right\}\frac{\partial\overline{\phi_1}}{\partial x_\beta}. \qquad [50]$$

Equations [38] and [50] may constitute a practical set of conservation equations of turbulence in gas–liquid two-phase flow. However, in order to close these equations and to make the calculation of turbulence feasible, one needs diffusion approximations for turbulent energy and dissipation and an assumption for the turbulent viscosity, μ_t. In single-phase flow, the turbulent viscosity is given in terms of turbulent energy and dissipation:

$$\mu_c = C\frac{K^2}{\epsilon}. \qquad [51]$$

This relation is based on the consideration of mixing length in single-phase flow turbulence. However, in two-phase flow, there exist two mixing lengths, one related to the size of the eddy and the other related to the size of the interfacial configuration (or inverse of the interfacial area concentration). Both mixing lengths are considered to be strongly related to turbulent energy and dissipation in gas–liquid two-phase flow. Under these circumstances, the validity of [51] should be more carefully examined.

At present, the knowledge about two-phase flow turbulence is not sufficient to make further comment about the modelling of turbulent viscosity in two-phase flow. However, in order to satisfactorily predict two-phase flow turbulence, it is essential to accumulate further experimental knowledge and improve the modelling of two-phase flow turbulence.

6. CONCLUSIONS

Based on the local instant and averaged formulations of gas–liquid two-phase flow, the conservation equations of mass and momentum for fluctuating terms were derived for incompressible two-phase flow without phase change.

Using these conservation equations, the conservation equation of turbulent energy was derived. It was shown that in this equation, interfacial transport terms of turbulent energy appeared. Detailed discussions on these transport terms were presented. As a result, it was shown that these interfacial transport terms were composed of turbulence generation due to the relative velocity between the phases and the exchange between turbulent and surface energy. Based on several approximations and assumptions, practical forms of the turbulent energy conservation equations were presented.

Also derived was the conservation equation of turbulent dissipation in gas–liquid two-phase flow. This equation is much more complicated and many more interfacial transport terms appeared compared with the turbulent energy conservation equation. Under several approximations and assumptions, the interfacial transport terms of turbulent dissipation were found to be mainly composed of two terms which include derivatives of pressure and velocity fluctuations at the interface. With some approximations, a practical form of the conservation equation of turbulent dissipation was presented.

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